



# LESSON 2. THE ENERGY BALANCE AND THE PENMAN- MONTEITH EQUATION

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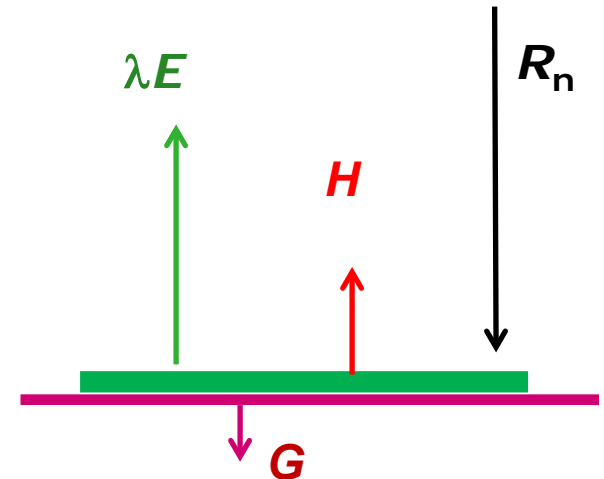
# The energy balance

- Derived from surface energy balance ( $\text{Wm}^{-2}$ ):

$$R_n = G + \lambda E + H$$

- $R_n$ : net radiation (shortwave + longwave)
- $G$ : ground/soil heat flux
- $\lambda E$ : latent heat flux for evaporation
- $H$ : sensible heat flux

$R_n$  is the source of energy,  $G$ ,  $H$  and  $\lambda E$  are the sinks of energy



*This is a 1-D (vertical) model concept*

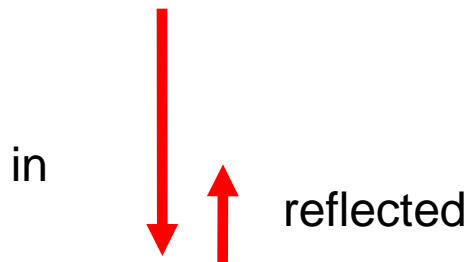


# Net shortwave radiation

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- Shortwave radiation from the **sun** and **sky** ( $0.4 - 2.5 \mu\text{m}$ )
- Reflection coefficient: albedo (or  $\alpha$ )

$$R_{s,n} = R_s^\downarrow - R_s^\uparrow = R_s^\downarrow - \alpha R_s^\downarrow = (1 - \alpha) R_s^\downarrow$$



## Albedo values

|             |        |
|-------------|--------|
| Water       | 4-8%   |
| Gray soils  | 15-25% |
| Green grass | 15-25% |
| Forest      | 10-15% |
| Fresh snow  | 80-90% |

# Net shortwave radiation

$$R_{n,l} = (1 - \varepsilon_s)R_{li} - R_{lo}$$

Incoming from the sky

with Stefan Boltzman's equation, For clear sky

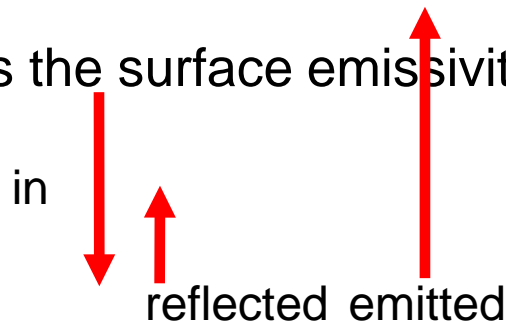
$$R_{ldc} = \varepsilon_{ac} \sigma T_a^4$$

With an empirical equation for the atmospheric emissivity  $\varepsilon_{ac}$

Emitted by the surface

$$R_{lu} = \varepsilon_s \sigma T_s^4$$

Where  $\varepsilon_s$  is the surface emissivity



# Sensible heat flux

- And resistance driven vertical vapour transport in the surface layer

$$H = \rho c_p \cdot \frac{T_0 - T_a}{r_a}$$

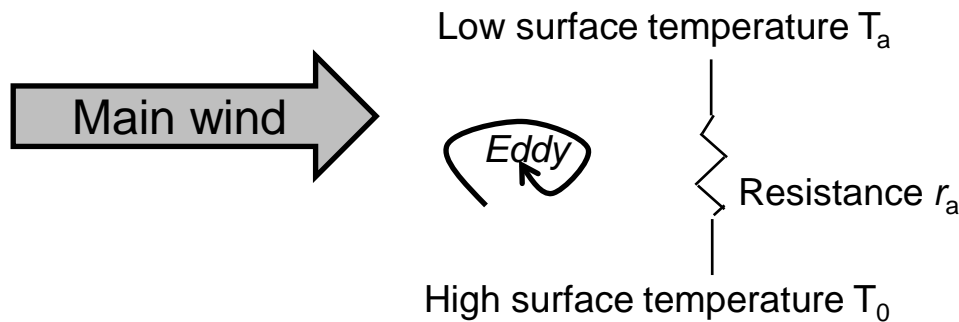
$e_s$ : vapour pressure at the surface (hPa)

$e_a$ : vapour pressure at some height  $z$  (hPa)

$r_a$ : aerodynamic resistance between surface and  $z$  ( $\text{sm}^{-1}$ )

$\rho$ : specific mass of air ( $\text{kg m}^{-3}$ )

$c_p$ : specific heat of air ( $\text{J kg}^{-1} \text{K}^{-1}$ )



# Latent heat flux

- For latent heat: extra resistance due to stomata and soil pores

$$\lambda E = \frac{\lambda \rho}{\gamma} \cdot \frac{e_s - e_a}{r_a + r_c}$$

$e_s$ : vapour pressure at the surface (hPa)

$e_a$ : vapour pressure at some height  $z$  (hPa)

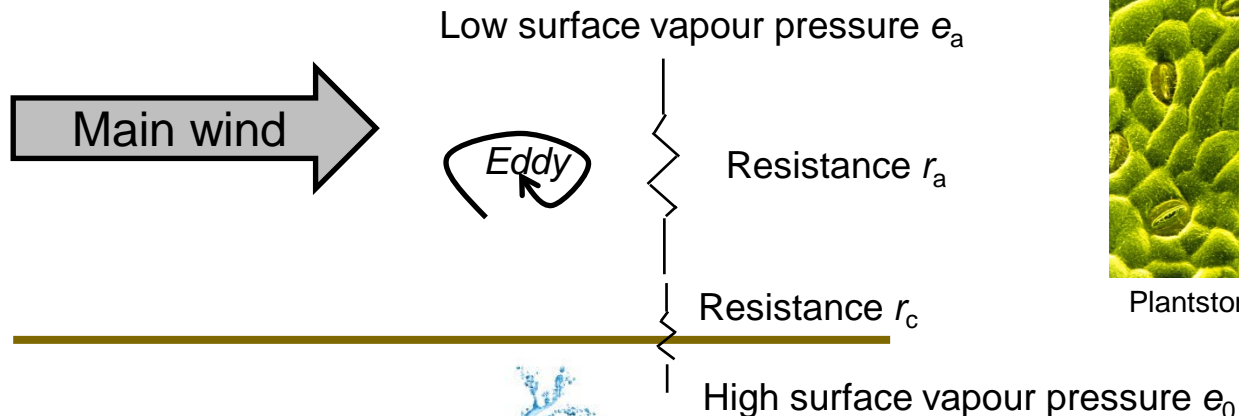
$r_a$ : aerodynamic resistance between surface and  $z$  ( $\text{sm}^{-1}$ )

$r_c$ : surface resistance between stomata/pores and surface ( $\text{sm}^{-1}$ )

$\lambda$ : latent heat for evaporation ( $\text{J kg}^{-1}$ )

$\rho$ : specific mass of air ( $\text{kg m}^{-3}$ )

$\gamma$ : psychrometer constant ( $\text{hPa K}^{-1}$ )



Plantstomata.worldpress.com

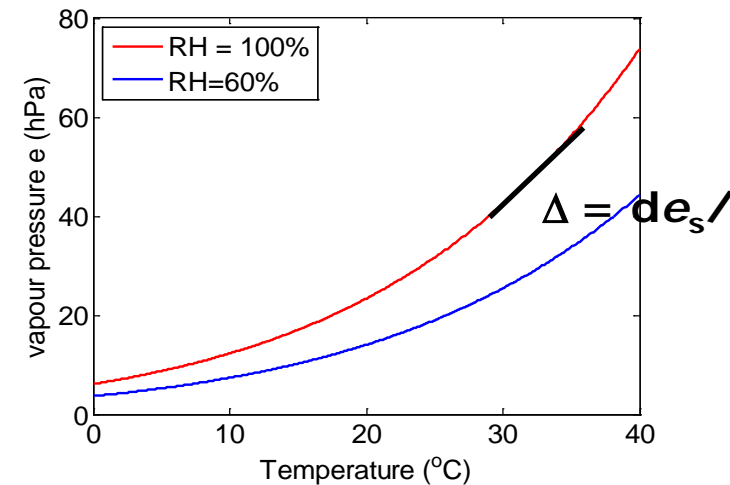
# Penman-Monteith equation

Approximate:

$$e_0 = e_s(T_0) \approx e_s(T_a) + \Delta \cdot [T_0 - T_a]$$

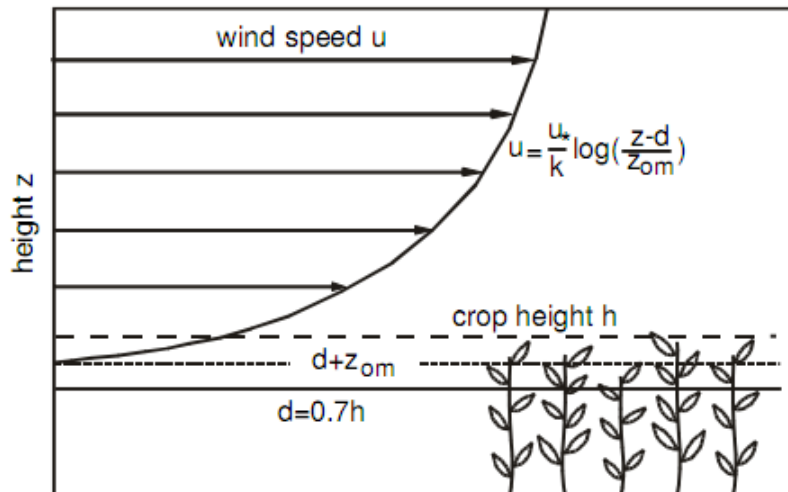
And combining the equations in the previous slides gives the equation of Penman-Monteith (Monteith 1965, Rijtema, 1965):

$$\lambda E = \frac{\Delta [R_n - G] + \frac{\rho c_p}{r_a} [e_s(T_a) - e_a]}{\Delta + \gamma \left[ 1 + \frac{r_c}{r_a} \right]}$$



# Calculation of aerodynamic resistance

- Vapour and heat transported by eddies
- Resistance depends on (1) wind speed and (2) vertical gradient of horizontal wind speed with height (the higher this gradient, the higher the resistance)
- Vertical profile of wind speed estimated from vegetation height and density and atmospheric stability







THE END  
THANK YOU

