Integrated approach to predict confidence of GPS measurement

Massoud Sharif [a], A. Stein [a], Ernst M. Schetselaar [b]
[a] Department of Earth Observation Sciences, sharif@itc.nl, stein@itc.nl,
[b] Department of Earth Systems Analysis, schetselaar@itc.nl,
International Institute for Geo-Information Science and Earth Observation (ITC), PO Box 6, 7500 AA Enschede, The Netherlands.

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ABSTRACT

Code measurement hand-held GPS serves a fast growing user community involved in navigation and data collection for field data collection, tourism, various public services and commercial applications. Most handheld GPS code receivers, however, do not provide instantaneous and intuitive information on the positional error during measurement, which may be important to compromise data accuracy against time efficiency in daily field practice. Herein we describe an experiment on how to assess the positional error of handheld code receivers and discuss a method that improves on the reporting of the positional error to the user in the field when reference points are available. In the experiment, the x and y coordinates of 100 reference points were used to assess the positional error of a handheld GPS code receiver. The coordinates of the points were computed from DGPS carrier phase measurements, and assumed to be error free. The error distribution in x and y for 1, 3, 5 and 10 minute averaging periods were computed to generate a figure of merit that is intuitive and informative to a non-expert GPS user. Results obtained are well within the range of the theoretical accuracy that can be expected for single measurements code receivers.

INTRODUCTION

Low cost handheld GPS receivers have become increasingly popular to a variety of users providing position, velocity and navigation information under all weather conditions. Most of these users need to know their position relative to a mapping base or real world topographic feature in real time in order to navigate, find, record or map geographic features. The users of a handheld GPS receiver, however, have only limited means to assess the accuracy of the GPS measurements in the field and hence cannot instantaneously obtain reliable information on the quality of GPS measurements and its variability.

The manuals of most GPS units provide a rough theoretical estimate of the horizontal and vertical accuracies. In addition, many units report statements of probable error, such as estimated position error (EPE) and figure of merit (FOM). The EPE is a scalar indicating the precision of the receiver based on the deviation of the measurements from the mean of the measurement. The FOM is a more mysterious quantity that uses in addition to the statistics of the measurements, additional information held confidential by the manufacturers of the code receivers, but which probably in addition to statistics on the GPS measurements uses either a theoretical or empirical error model to predict the accuracy of a code receiver.

Hence, users of single handheld GPS units do not have instantaneous access to explicit accuracy reports of the GPS measurements. This is, besides the lack of explanation on how the accuracy indicators are computed, also due to the lack of confidence intervals given in the displayed EPE reports. For example it is often not known whether the EPE is based on a 50 % confidence interval, the route mean square error (68 % interval) or any other confidence interval. Given the above, the non-expert GPS user has no effective means to obtain insight in the accuracy of the handheld GPS equipment using it at a particular location and time. To overcome this limitation, the user needs to be able to conduct field experiments to assess the accuracy of the GPS unit in the area of interest and generate accuracy reports that are unambiguous and intuitive to understand. Ultimately these reports may, together with the statistics of the GPS measurements acquired at a given location and time, provide a basis for empirical predictions of the accuracy (e.g. FOM) of a particular instrument when no reference points are available.

In this paper we discuss an experiment that can be used to assess and generate reports of the accuracy of single handheld GPS units. The focus of this paper is on the accuracy of the horizontal position information, although accuracy assessment has a bearing on time, horizontal and vertical position, and velocity as well. Our analysis refers to the assessment to what extent the measured position of a particular point on the surface of the Earth conforms with respect to the true position of that point.

MATERIALS AND METHODS.

When a handheld GPS code receiver is used for positioning, one can get a rough indication about the accuracy by referring to the theoretical figure of the GPS error model, as presented in table 1.

One can see that the horizontal accuracy of a single handheld receiver is 10.2 m, and accuracy of differential measurements would be 3.1 m. The table is based on 1 σ corresponding to a 68% confidence interval. Note that the user does not have any assurance to obtain this accuracy in practice. Hence, the need for an empirical quality measure computed during GPS measurements at a particular time and place. In the following we provide the error analysis needed to compute the positional error of a code receiver on the basis of the coordinates of known reference points.

Error source	Bias	Random	Total	DGPS
Ephemeris data	2.1	0.0	2.1	0.0
Satellite clock	2.0	0.7	2.1	0.0
Ionosphere	4.0	0.5	4.0	0.4
Troposphere	0.5	0.5	0.7	0.2
Multipath	1.0	1.0	1.4	1.4
Receiver noise	0.5	0.2	0.5	0.5
UERE, rms*	5.1	1.4	5.3	1.6
Filtered UERE, rms	5.1	0.4	5.1	1.5
VDOP= 2.5			12.8	3.9
HDOP= 2.0			10.2	3.1

Table 1 Standard GPS error model based on 1 σ (corresponding to a 68% confidence interval)- L1 C/A (no selected availability)

Where DGPS is Differential GPS, UERE, is the user equivalent range error that sums the various error components, VDOP is vertical dilution of precision, and HDOP is horizontal dilution of precision. The table shows errors of Pseudorange measurements (Ephemeris data, Satellite clock error, Ionospheric error, Tropospheric error, Multipath, and the Receiver noise).

1.ASSESSING THE POSTIONAL ERROR USING REFERENCE POINTS

The accuracy of GPS observation can be estimated in various ways. One method is to define confidence regions are ellipsoids (in 3D space), or ellipses (in 2D space), such that their volume or area contains the true value within a preselected level of probability. A useful scalar for assessing the error often displayed on handheld units is the dilution of precision (DOP), being a factor of reduction of precision (in function of satellite constellation at the moment of observation). In the following we will show how DOP and confidence regions can be derived from the covariance matrix of the unknown vector of the three geocentric coordinates and travel time (e.g. x, y, z, dτ) to be determined from the GPS observations (Wells 1986)

A pseudo-range observation equation, using code, for one satellite can be written as:

$$P^{Sat} = c.d\tau^{Sat} = \rho^{Sat} + c.(dT_{Rec} - dt^{Sat}) + d^{Sat}_{Ion} + d^{Sat}_{Trop} \quad (1)$$

Where c = 299792458 m/sec (the speed of light in a vacuum), dT_{rec} the receiver clock error, dt^{sat} the satellite clock error, d_{ion} ionospheric error, d_{trop} tropospheric error and

$$\rho = \sqrt{(x^{Sat} - x_{Rec})^2 + (y^{Sat} - y_{Rec})^2 + (z^{Sat} - z_{Rec})^2} \quad (2)$$

the true range receiver-satellite, ignoring atmospheric effects

For more than one satellite, we obtain a set of equations that is over determined:

$$P^1 = c.d\tau^1 = \rho^1 + c.(dT_{Rec} - dt^1) + d^1_{ion} + d^1_{trop}$$

$$P^2 = c.d\tau^2 = \rho^2 + c.(dT_{Rec} - dt^2) + d^2_{ion} + d^2_{trop}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$P^n = c.d\tau^n = \rho^n + c.(dT_{Rec} - dt^n) + d^n_{ion} + d^n_{trop}$$

These GPS observation equations can be written in matrix notation as:

$$A * X = B + V \quad (4)$$

Where *X* is the unknown vector matrix containing (z, y, z, dτ), *A* is the coefficient matrix of the unknowns (number of columns = number of unknown parameters, number of rows = number of visible satellites), *B* is the matrix of the known values from reference points, and *V* is the error matrix. Applying generalized least squares; an estimate of the unknown vector matrix is obtained as:

$$X = (A^T W A)^{-1} A^T W B \quad (5)$$

Where *W* is the weight coefficient matrix. In addition estimated observation errors are obtained as:

$$V = A((A^T W A)^{-1} A^T W B) - B. \quad (6)$$

And a-posteriori variance factor is defined as:

$$\sigma_0^2 = \frac{V^T W V}{n - k} \quad (7)$$

where σ_0 is the mean square error of unit weight, n is the number of observations, k is the minimum number of observation required, and $n-k$ is the number of redundant observations.

The variance-covariance matrix of the unknowns is defined as

$$C_x = \sigma_0^2 (A^T W A)^{-1} \quad (8)$$

$$C_x = \sigma_0^2 \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} & \sigma_{x\tau} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} & \sigma_{y\tau} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} & \sigma_{z\tau} \\ \sigma_{\tau x} & \sigma_{\tau y} & \sigma_{\tau z} & \sigma_{\tau\tau} \end{bmatrix} \qquad (9)$$

Where the diagonal elements of the matrix refer to the are σ_{xx} the variance of x, σ_{yy} variance of y, σ_{zz} variance of z, and $\sigma_{\tau\tau}$ variance of time, and off-diagonal elements are covariances. Various dilutions of precision factors can now be computed:

The horizontal dilution of precision is defined as:

$$HDOP = \frac{\sqrt{\sigma_{xx} + \sigma_{yy}}}{\sigma_0}, \qquad (10)$$

and the vertical dilution of precision as:

$$VDOP = \frac{\sqrt{\sigma_{zz}}}{\sigma_0} \qquad (11)$$

The semi-axis of the confidence regions are related to the eigenvalues of this matrix (Vanicek, P. and E.J. Krakiwsky, , 1986)

2. DATA COLLECTION AND VALIDATION

To practically verify the GPS error model, and estimate the positional error of the measurements of ground control points using a GPS code receiver, the discrepancies between receiver coordinates and reference coordinates are computed in a study area around the ITC building in Enschede, The Netherlands using a Garmin map 76s code receiver and a Leica 300 differential carrier phase instrument. The coordinates of the points obtained by carrier phase measurement were used as reference, to compute errors of the code measurements.

3. COORDINATE SYSTEMS.

For the analysis 3 different coordinate systems were used:

-WGS 84, which is the common GPS coordinate system.
-RD and NAP, Dutch coordinate system, used in The Netherlands.
- Local (plane tangential) system,

The transformation parameters used to transform data from WGS84 to RD and NAP, used in phase observation, are different to that applied by Garmin code receivers. Therefore the geodetic WGS84 coordinate (φ, λ, h) is converted To X, Y, Z geocentric coordinate, and discrepancies ΔX, ΔY, ΔZ are computed.

To be interpretable for practical use, geocentric discrepancies ΔX, ΔY, ΔZ are transformed into a plane tangent to the earth surface, at the centre of gravity of the study area, the local coordinate system (ΔN, ΔE, ΔH).

The following equations were applied for the transformation of discrepancies from ΔX, ΔY, and ΔZ to the local coordinated system in the plane surface, which is shown (ΔN, ΔE, ΔH)

$$\Delta E = -\Delta X \cdot \sin\lambda + \Delta Y \cdot \cos\lambda$$

$$\Delta N = -\Delta X \cdot \cos\lambda \cdot \sin\varphi - \Delta Y \cdot \sin\lambda \cdot \sin\varphi + \Delta Z \cdot \cos\varphi \qquad (11)$$

$$\Delta H = \Delta X \cdot \cos\lambda \cdot \cos\varphi + \Delta Y \cdot \sin\lambda \cdot \cos\varphi + \Delta Z \cdot \sin\varphi$$

Where, φ refers to the geodetic latitude of a target point and λ, is defined as the geodetic longitude of a target point.

4. RESULTS

The average of discrepancies **μ** represents the deterministic error, and **σ** indicates the stochastic error. Four different observation times (1, 3, 5, and 10 minutes) were used, applying a Garmin map76s handheld receiver. The following tables show the result in the local coordinate system.

Discrepancy	μ	σ^2	σ	RMSE
ΔX	-0.78	4.55	2.13	2.24
ΔY	-1.04	5.67	2.38	2.56
Cov.		-0.99		
Correl. oeff.		-0.19		
Plani. MSE		3.40		

Table 2. Statistics of Discrepancy of 1-minute observations:

Discrepancy	μ	σ^2	σ	RMSE
ΔX	-0.72	3.02	1.74	1.87
ΔY	-1.04	5.14	2.27	2.48
Cov		-0.96		
Correl. coeff.		-0.25		
Plani. RMSE		3.11		

Table 3. Statistics of Discrepancy of 3-minutes observations:

Discrepancy	μ	σ^2	σ	RMSE
ΔX	-0.46	2.12	1.46	1.52
ΔY	0.82	5.60	2.38	2.32
Cov.		-0.17		
Correl.coeff.		-0.06		
Plani. RMSE		2.77		

Table 4. Statistics of Discrepancy of 5-minutes observations

Discrepancy	μ	σ^2	σ	RMSE
X	-0.28	1.89	1.38	1.40
Y	-0.54	2.37	1.55	1.63
Cov.	-0.27			
Correl.coeff.	-0.13			
Plan. RMSE	2.15			

Table 5. Statistics of Discrepancy of 10-minutes observations: -

Tables 2, 3, 4, and 5 show that the deterministic error, as well as the stochastic error; are decreasing as a function of increasing the observation times (1_{min}, 3_{min}, 5_{min}, 10_{min})

4.1 Averaging Positionin

To investigate the influence of time averaging, results of 4 different acquisition times, namely; 1min, 3min, 5min, and 10min, are presented in the following tables for comparison.

Averaging Time	RMSE	σ
1 MINUTE	3.41	3.20
3 MINUTES	3.11	2.86
5 MINUTES	2.78	2.62
10 MINUTES	2.15	2.06

Table 6. Effect of averaging (in meters)

Results of the table 6 show an increase in accuracy on averaging over longer periods, as expected.

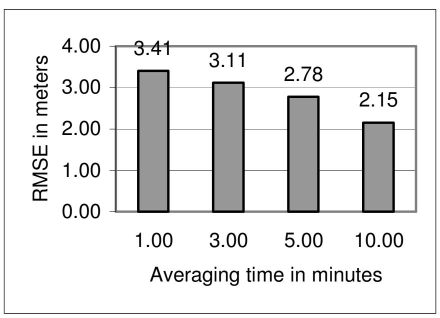


Figure 1 RMSE Vs Observation Time

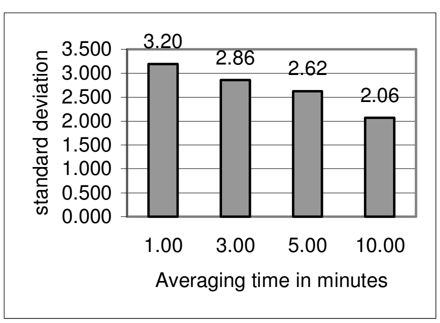


Figure 2 Standard Deviation Vs Observation Time

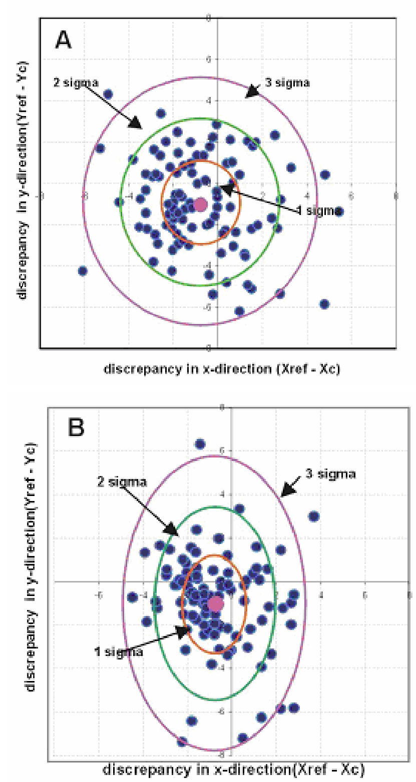
Figures 1, and 2 show a decrease in RMSE, respectively standard deviation as a function of averaging time.

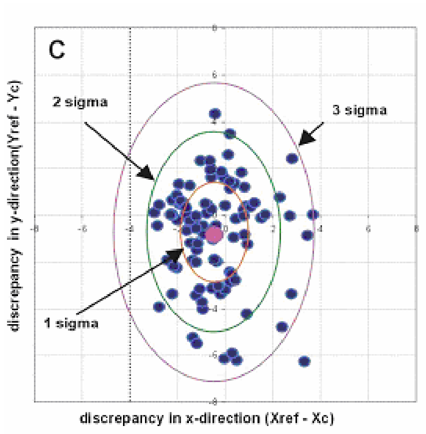
5. TESTING DISCREPANCIES AT DIFFERENT CONFIDENCE LEVELS

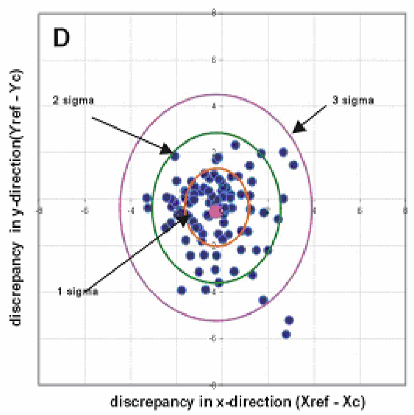
Estimation of the confidence interval of the errors requires estimation of statistical parameters, followed by evaluation of the statistical distribution model of the population parameters. A large number of samples are needed to have a valid conclusion. It was further assumed that the discrepancies follow a normal distribution, N (μ, σ). Statistical analysis of discrepancies consist of testing: H_0 : the individual discrepancy belongs to the population error, against, and H_1 : the individual discrepancy is not a member of this population:

To compute confidence intervals the values $\overline{X} \pm z_c \sigma_{\overline{x}}$ are used, where, z_c is the standard normal confidence level, and, $\overline{X}$ and $\sigma_{\overline{x}}$ are the sample mean and standard deviation of the sample, respectively.

Discrepancy scatter plot, for time-averaged positions at 1, 3,5, and 10 minutes, are presented in figure 3 (A, B, C, D) respectively.







Planimetric discrepancy   Mean Planimetric discrepancy

Figure 3 Discrepancy scatter plots, for time-averaged positions, A) 1 minute averaging, B) 3 minutes averaging, C) 5 minutes averaging, D) 10 minutes averaging.

Practical discrepancy outliers at 1σ, 2σ, and 3σ, for 1, 3, 5, and 10 minutes are presented in table 7

z_c	*Probability%*	*Percentage of outliers*								
		Theoretical%	*Practical%*							
			1m		3m		5m		10m	
			X	Y	X	Y	X	Y	X	Y
1 σ	*68.27*	*31.73*	34	28	40	23	35	31	35	25
2 σ	*95.45*	*4.55*	7	8	5	8	4	7	7	9
3 σ	*99.73*	*0.27*	0	1	0	3	0	0	0	2

Table 7. Practical discrepancy outliers for 1, 3, 5, and 10 minutes

From this table we conclude that all Y discrepancies are satisfying the test of H_0 against H_1, at 68% confidence and for all 4 different averaging time, but in X direction the number of practical outliers are larger than the theoretical outliers for all averaging times. At 95.45% confidence all the practical outliers are larger than the theoretical outliers, except the X outlier at 3 minutes averaging time. At 99.97% confidence all discrepancies in X direction are satisfying the test of H_0 against H_1, in Y direction they are all larger than the theoretical outliers, except for 5 minutes averaging time.

6. DISCUSSION

When reference points are available the user can perform code measurements to estimate the bias and random error, and use the results as a figure of merit for the measurements in the area of interest. The reference points may be obtained from national mapping authorities or from precise carrier-phase instruments measuring in differential mode. Preferably the coordinates of the references points should be defined in WGS84 to eliminate the effects of differences in datum transformation parameters as applied by the mapping authorities and those used by the code receiver handheld units. Averaging can reduce the effect of random error and provides an indication of the systematic errors. The results of our averaging experiment show that the standard deviation is reduced with 65 percent while the bias is reduced with 47 percent from 1 to 10 minutes averaging. If this difference in the reduction of the accuracy and precision would be significant, it implies that the EPE reports become better predictors for the accuracy with increasing averaging time. This initial result, however, needs to be confirmed and we recommend users to repeat measurements on different hours of the day and monitor the effects of variations of constellation. In addition, one may refer to HDOP values, which may be provided by some instruments. The mean, RMSE of HDOP, and the mean number of available satellites can be used to compute an average for the expected errors. In table 8 these parameters are given for the present experiment.

HDOP reported by handheld code receiver	
Mean HDOP	2.23
RMS HDOP	0.31
Mean no. Satellite	6

Table 8. Number of satellites, HDOP values and RMS of HDOP for handheld code receiver.

If no reference points are available, the averaged position is often used as an approximation to the actual position (Wilson, 2002). This will generally lead to an underestimation of the predicted error because the bias is ignored. It is important, therefore to convey this to the user and to carefully assess the range in systematic errors in the accuracy assessment experiments of a handheld unit. Alternatively some units provide statements on probable errors, such as the horizontal estimated position error (EPE) or figure or merit (FOM) for average positions (DePriest, 2002). FOM estimates are usually different for each company and computation is held confidential so that there are no means available to evaluate their robustness.

Regardless reference points are available or not, the user of handheld units currently lacks information that allows one to make an instantaneous judgement on the quality of GPS data during measurement, on which basis adequate decision can be taken in the field. With the current trend of customisable mobile GIS applications, however, it has become possible to access the various GPS protocols, such as NMEA, which allows users to program their own accuracy assessment and figure of merit reports in a customized mobile GIS applications. Accuracy experiments to assess the accuracy can then be normalized for HDOP statistics and averaging time to provide a basis for an error prediction that is better controlled by the user. The following provide some suggestions on how to further enhance the FOM reports currently employed by non-expensive handheld units:

1. Directional dependencies of the error can currently not be assessed. This could be important in applications where existing waypoints need to be relocated in the field or where the mapping of directional features, such as unit boundaries need to be validated. Information on the anisotropy in the distribution of measured GPS positions, allows the user to assess directional variation of the error.

2. In the current handheld system the user has only access to one percentile in the probability distribution of the error estimate (usually the CEP 50 or RMS 68 percentiles). In most cases the field surveyor is not that interested in such optimistic error estimates where there is still a probability of 50 to 47 percent that a particular measurement falls outside the specified range. More conservative error estimates based for example on the 95 or 99 percentiles, are probably more useful to the average user, because such estimates minimizes the risk for the GPS measurement to be outside the reported error estimate. Ideally the user should have access to the modelled error distribution during measurement.

3. The influence of outliers on the error estimate cannot be assessed or outliers above a user-defined threshold cannot be rejected on the basis of their associated poor DOP value. Functions to reject outliers would greatly improve the accuracy of time-averaged GPS data.

7. CONCLUSIONS

In this paper we discussed methods for the assessment and reporting the error of handheld code receivers. The figure of merit (FOM) and estimated position error reports (EPE) of the code receivers are often misleading, because the effects of bias can not be considered. Non-expert users may not bet aware of this limitation and may rely on overoptimistic error estimates. The following suggestions are meant to overcome these limitations:

1. Non-expert users should be guided by GPS handheld vendors in conducting simple experiments in their study area, to determine the accuracy of their instrument when reference points are available.

2.The effects of variations of the constellation during the day, the masking effects and the effect of averaging time on the accuracy assessment should not be underestimated in such experiments.

3.Our experiment suggests that the practical confidence is not always in agreement with the theoretical (estimated) confidence. For the 99% confidence level, for example, the percentage of outliers are higher then estimated, probably as a result of the relatively small sample size.

4.The RMS of the averaged planimetric position, together with HDOP at the moment of the observation seems to be a realistic FOM for handheld receivers.

5.More FOM reporting functions, such as scatter plots, time-average graphs, DOP as a function of time and probability graphs are needed for the demanding users of code receivers.

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